

1 Strangeness constraint in Heavy Ion Collisions

In order to compare our lattice calculations to experimental results from heavy ion collisions we want to expand in the chemical potential but keep the net-strangeness number equal to zero. Let us consider the Taylor expansion for the pressure in terms of the quark chemical potentials $\hat{\mu}_q \equiv \mu_q/T$ and $\hat{\mu}_s \equiv \mu_s/T$,

$$\frac{\Delta p}{T^4} = \sum_{i,j} c_{i,j} \hat{\mu}_q^i \hat{\mu}_s^j, \quad \text{with} \quad c_{i,j} \equiv \frac{1}{i!j!} \frac{\partial^i}{\partial \hat{\mu}_q^i} \frac{\partial^j}{\partial \hat{\mu}_s^j} \frac{1}{V T^3} \ln Z \quad . \quad (1)$$

Note that the faculties are included in the definition of the expansion coefficients. Due to symmetries of the partition function we have

$$c_{i,j} = 0 \quad \forall (i+j) \text{ odd}. \quad (2)$$

Using this expansion coefficients one can write the strange quark density as function of $\hat{\mu}_q$ and $\hat{\mu}_s$

$$n_s(\hat{\mu}_q, \hat{\mu}_s) = \sum_{i,j} (j+1) c_{i(j+1)} \hat{\mu}_q^i \hat{\mu}_s^j \quad (3)$$

$$= c_{11} \hat{\mu}_q + 2c_{02} \hat{\mu}_s \quad (4)$$

$$+ c_{31} \hat{\mu}_q^3 + 2c_{22} \hat{\mu}_q^2 \hat{\mu}_s + 3c_{13} \hat{\mu}_q \hat{\mu}_s^2 + 4c_{04} \hat{\mu}_s^3 \quad (5)$$

$$+ c_{51} \hat{\mu}_q^5 + 2c_{42} \hat{\mu}_q^4 \hat{\mu}_s + 3c_{33} \hat{\mu}_q^3 \hat{\mu}_s^2 + 4c_{24} \hat{\mu}_q^2 \hat{\mu}_s^3 + 5c_{15} \hat{\mu}_q \hat{\mu}_s^4 + 6c_{06} \hat{\mu}_s^5 \quad (6)$$

$$+ \dots \quad (7)$$

If we constrain n_s to be zero, the strange quark chemical potential $\hat{\mu}_s$ becomes a function of the light quark chemical potential $\hat{\mu}_q$. We have

$$\hat{\mu}_s(\hat{\mu}_q) = \sum_i d_i \hat{\mu}_q^i = \left(-\frac{c_{11}}{2c_{02}} \right) \hat{\mu}_q + \left(\frac{2c_{04}c_{11}^3 - 3c_{02}c_{11}^2c_{13} + 4c_{02}^2c_{11}c_{22} - 4c_{02}^3c_{31}}{8c_{02}^4} \right) \hat{\mu}_q^3 + \mathcal{O}(\hat{\mu}_q^5) \quad . \quad (8)$$

When applying this to the pressure expansion, one gets

$$\frac{\Delta p}{T^4} = \sum_i \hat{c}_i \hat{\mu}_q^i \equiv \sum_{i,j} c_{ij} \hat{\mu}_q^i \left(\sum_k d_k \hat{\mu}_s^k \right)^j \quad (9)$$

$$= \left(c_{20} - \frac{c_{11}^2}{4c_{02}} \right) \hat{\mu}_q^2 + \left(c_{40} + \frac{c_{04}c_{11}^4}{16c_{02}^4} - \frac{c_{11}^3c_{13}}{8c_{02}^3} + \frac{c_{11}^2c_{22}}{4c_{02}^2} - \frac{c_{11}c_{31}}{2c_{02}} \right) \hat{\mu}_q^4 + \mathcal{O}(\hat{\mu}_q^6) \quad . \quad (10)$$

For the strange quark fluctuation we obtain

$$\frac{\chi_s}{T^2} = \sum_i \hat{s}_i \hat{\mu}_q^i = 2c_{02} + \left(2c_{22} - \frac{3c_{13}c_{11}}{c_{02}} + \frac{3c_{04}c_{11}^2}{c_{02}^2} \right) \hat{\mu}_q^2 \quad (11)$$

$$+ \left(2c_{42} + \frac{3c_{11}^2}{c_{02}^2} + \frac{15c_{06}c_{11}^4}{8c_{02}^4} - \frac{5c_{11}^3c_{15}}{2c_{02}^3} - \frac{3c_{11}c_{33}}{c_{02}} \right. \quad (12)$$

$$\left. - \frac{3c_{04}c_{11}(2c_{04}c_{11}^3 - 3c_{02}c_{11}^2c_{13} + 4c_{02}^2c_{11}c_{22} - 4c_{02}^3c_{31})}{2c_{02}^5} \right) \quad (13)$$

$$+ \frac{3c_{13}(2c_{04}c_{11}^3 - 3c_{02}c_{11}^2c_{13} + 4c_{02}^2c_{11}c_{22} - 4c_{02}^3c_{31})}{4c_{02}^4} \right) \hat{\mu}_q^4 + \mathcal{O}(\hat{\mu}_q^6) \quad . \quad (14)$$

2 The flavor symmetric case

Another interesting case is the flavor symmetric case. Now we constrain the quark number densities of light and strange quark to be equal, i.e.

$$n_s(\hat{\mu}_q, \hat{\mu}_s) \equiv \frac{1}{2} n_q(\hat{\mu}_q, \hat{\mu}_s) \quad . \quad (15)$$

Solving for $\hat{\mu}_s$, order by order in $\hat{\mu}_q$, we get

$$\hat{\mu}_s(\hat{\mu}_q) = \sum_i d_i \hat{\mu}_q^i = \left(\frac{c_{20} - c_{11}}{2c_{02} - c_{11}/2} \right) \hat{\mu}_q \quad (16)$$

$$+ \left(\frac{64c_{04}(c_{11} - c_{20})^3}{(-4c_{02} + c_{11})^4} + \frac{24c_{13}(c_{11} - c_{20})^2}{(-4c_{02} + c_{11})^3} + \frac{8c_{22}(c_{11} - c_{20})}{(-4c_{02} + c_{11})^2} + \frac{2c_{31}}{(-4c_{02} + c_{11})} \right) \hat{\mu}_q^3 + \mathcal{O}(\hat{\mu}_q^5) \quad . \quad (17)$$

Again, we plug this solution into the Taylor expansion for pressure and strange quark fluctuation and get

$$\frac{\Delta p}{T^4} = \sum_i \hat{c}_i \hat{\mu}_q^i \equiv \sum_{i,j} c_{ij} \hat{\mu}^i \left(\sum_k d_k \hat{\mu}_s^k \right)^j \quad (18)$$

$$= (c_{20} + c_{11}d_1 + c_{02}d_1^2) \hat{\mu}_q^2 + (c_{40} + c_{31}d_1 + c_{22}d_1^2 + c_{13}d_1^4 + c_{04}d_1^4 + c_{11}d_2 + 2c_{02}d_1d_2) \hat{\mu}_q^4 + \mathcal{O}(\hat{\mu}_q^6) \quad , \quad (19)$$

and

$$\frac{\chi_s}{T^2} = \sum_i \hat{s}_i \hat{\mu}_q^i = 2c_{02} + (2c_{22} + 6c_{13}d_1 + 12c_{04}d_1^2) \hat{\mu}_q^2 \quad (20)$$

$$+ (2c_{42} + 6c_{33}d_1 + 12c_{24}d_1^2 + 20c_{15}d_1^3 + 30c_{06}d_1^4 + 6c_{13}d_2 + 24c_{24}d_1d_2) \hat{\mu}_q^4 + \mathcal{O}(\hat{\mu}_q^6) \quad . \quad (21)$$